A formal model for epistemic interactions

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Abstract. The conceptual world of AI is inhabited by a number of epistemic puzzles whose role is to provide a test harness environment for various methods and algorithms. In our paper we focus on those puzzles in which agents either collaborate or compete with one another in order to adopt their epistemological situations to their environment. Our goal is to devise a formal model for epistemic interactions and a family of reasoning mechanisms that would solve those puzzles. Once specified in the abstract manner, they are implemented in the Prolog environment.

Key words: knowledge representation, ontology, epistemic change

Introduction

The conceptual world of Artificial Intelligence (AI) is inhabited by a number of epistemic puzzles whose role is provide a test harness environment for various methods and algorithms. In our paper we will focus on those puzzles in which agents either collaborate or compete with one another in order to adopt their epistemological situations to their environment. In the puzzles at stake they exhibit certain behaviours by means of which they attempt to reach certain epistemic goals. Here is an exemplary list of such puzzles: the hats puzzle ([2]), the wisemen puzzle ([8]), Mr Sum and Mr Product puzzle ([8]), the muddy children puzzle ([1]). Our goal is to devise a family of reasoning mechanisms that would solve the puzzles. Once specified in the abstract manner, they will be implemented in the Prolog environment. As a working example we will use a well-known *hats puzzle*:

> Three people Adam, Ben and Clark sit in a row in such a way that Adam can see Ben and Clark, Ben can see Clark and Clark cannot see anybody. They are shown five hats, three of which are red and two are black. The light goes off and each of them receives one of the hats on his head. When the light is back on they are asked whether they know what the colours of their hats are. Adam answers that he doesn't know. Then Ben answers that he doesn't know either. Finally Clark says that he knows the colour of his hat. What colour is Clark's hat?

John McCarthy in [7] points out that AI methods of solving problems suffer unbalance between their two parts: epistemological and heuristic. The first one is a representation of the world which enables one to solve problems while the second one is a mechanism of finding solutions. McCarthy notes that most of the work is devoted to heuristic part.

In this paper we will pick up the task of developing the epistemological part of intelligence. Accepting the tenets of the Knowledge Representation paradigm, we believe that developing a general formal model of agents' epistemologies will provide a firm and universal basis for the algorithms we are up to.

In section 1 a general model of epistemic interactions is introduced and described. In section 2 we present an implementation of our model in Prolog. Then in section 3 a comparison of our model with other frameworks is given. At the end we discuss some directions of evolution of our model and its applications.

1 Towards a general model of epistemic interactions

Our model of epistemic interactions has two components: ontological and epistemological. The ontological part represents, in rather rough and ready way, the world our knowledge concerns. The epistemological part of the model represents the phenomenon of knowledge in its static and dynamic aspects.

We start to discuss the ontological component of the model with an analysis of the notion of "situation". A belief, as an intentional entity, refers to an external chunk of reality, which we call a *situation* (or state of affairs). So when I believe that Warsaw is the capital of Poland then this belief of mine concerns the situation *that Warsaw is the capital of Poland*, which situation is somehow part of the real world. In a general, the situation at stake may have any ontic structure. Thus, there are situations "in which" certain objects possess certain properties, situations "in which" certain objects participate in certain relations or processes, etc.

Let *ElemSit* be a set of elementary ontic (possible) situations. Briefly speaking, a situation is *elementary* if no situation is part of it. For instance *that Adam has a red hat* would be an elementary situation and *that both Adam and Ben have red hats* would not be an elementary situation.

In the set *ElemSit* we define the relation of compossibility. Intuitively, $x \parallel y$ means that a situation x may co-occur with situation y.[1] For example, if $x =$ *that Adam has a red hat*, $y =$ *that Ben has a red hat* and $z =$ *that Adam has a white hat*, then $x \parallel y$, $y \parallel z$ and $x \nparallel z$. For obvious reasons, the relation $\parallel$ is reflexive and symmetric in *ElemSit*, but is not transitive.

[1] It is our intention that the meaning of "may" remains within the domain of ontology. In other words, if $x \nparallel y$, then x cannot co-occur with y because of some ontological principle, where ontology is understood broadly enough to include laws of logic. However, the concept of ontology used here has more to do with the engineering understanding than with the philosophical tradition (see, for instance, [5]). So, effectively, relation $\parallel$ is relative to *an ontology*.

In what follows we will mainly deal with non-elementary situations (situations, for short), which will be represented as sets of elementary situations. We use the notion of situation instead of the notion of elementary situation because a lot of our beliefs do not concern elementary situations. Let $\emptyset \notin Sit \subseteq \wp(ElemSit)$ be a set of (possible) ontic situations.[2] In our example, that both Adam and Ben have red hats might be represented as the set $\{x, y\} \in Sit$. Given our understanding of the relation $\|$, the following condition is the case:

$$X \in Sit \rightarrow \forall y, z \in X y \parallel z. \tag{1}$$

We do not accept the opposite implication because we do not want to commit our model to such entities as the situation that Warsaw is the capital of Poland and π is an irrational number.

We can now define the notion of possible world:

$$X \in PossWorld \triangleq X \in Sit \wedge \forall Y (X \subset Y \rightarrow Y \notin Sit). \tag{2}$$

Let *Agent* be a set of epistemic agents, i.e. those agents that are capable of having beliefs and $Time = (t_1, t_2, \dots)$ be a sequence of moments. The *actual epistemic state* of an agent at a given moment will be representeded by a subset of $PossWorld$: $epist(a, t_n) \subseteq PossWorld$. Any such state collectively, so to speak, represents both the agent's knowledge and his or her ignorance. Due to its actual epistemic state, which is represented by a set $epist(a, t_n)$, and for every $X \in Sit$, agent a may be described (at t_n) according to the following three aspects:[3]

Definition 1. *Agent a knows at moment t_n that situation X holds (written: $K_{a,t_n}(X)$) iff $X \subseteq \bigcap epist(a, t_n)$.*

Definition 2. *Agent a knows at moment t_n that situation X does not hold (written: $\overline{K}_{a,t_n}(X)$) iff $X \cap (\bigcup epist(a, t_n)) = \emptyset$.*

Definition 3. *Agent a does not have any knowledge at moment t_n about situation X iff $\neg K_{a,t_n}(X) \wedge \neg \overline{K}_{a,t_n}(X)$.*

However, the puzzles we are dealing with do not presuppose that we know the actual epistemic state of a given agent. Thus, we extend the notion of actual epistemic state to the notion of possible epistemic state. A *possible epistemic state* of an agent represents a body of knowledge (resp. of ignorance) that the agent may exhibit given the ontic situation and epistemic capabilities of this agent. In our case, the possible epistemic states are determined by the relation of seeing (perceiving), other agents' announcements, and the agent's deductive capabilities.

A possible epistemic state of an agent at a given moment will be represented by the set $epist_i(a, t_n) \subseteq PossWorld$. One of the possible epistemic states is the

[2] As before, the specific content of *Sit* is determined by the ontology presupposed in a given domain (or puzzle).

[3] The definitions, and their extensions 4, 5, and 6 below, presuppose that the set *Sit* is closed under intersections: $X, Y \in Sit \wedge X \cap Y \neq \emptyset \rightarrow X \cap Y \in Sit$.

actual epistemic state, i.e. there exists i such that $epist(a, t_n) = epist_i(a, t_n)$. As before (for $Y \in Sit$), an agent may be described (at t_n) according to the following three aspects:

Definition 4. *Agent a knows in a possible epistemic state $X = epist_i(a, t_n)$ that (ontic) situation Y holds (written : $K_X(Y)$) iff $Y \subseteq \bigcap X$.*

Definition 5. *Agent a knows in a possible epistemic state $X = epist_i(a, t_n)$ that (ontic) situation Y does not hold (written : $\overline{K}_X(Y)$) iff $Y \cap (\bigcup X) = \emptyset$.*

Definition 6. *Agent a does not have any knowledge in a possible epistemic state $X = epist_i(a, t_n)$ about ontic situation Y iff $\neg K_X(Y) \wedge \neg\overline{K}_X(Y)$.*

We will use later the following auxiliary notions: $Epist(a, t_n)$ – a set of all possible epistemic states of agent a at t_n, $Epist(t_n)$ – a set of sets of possible epistemic states of all agents at t_n, $Epist$ – a set of sets of all possible epistemic states of all agents (from $Agent$) at all moments (from $Time$).

When $a \in Agent$, then "$\sim_a$" will represent the relation of epistemological indiscerniblity, which we treat as an equivalence relation. In general, the epistemological indiscernibility covers a number of epistemic constraints of agents. For example:

- (Because Adam does not see his head), he does not discern the situation in which he has a red hat from the situation in which has a white hat,
- (Because of his daltonism), Ben does not discern the situation in which the traffic signal is red from the situation in which the traffic signal is green,
- (Because of the thermostat's failure), the central heating system does not "discern" the situation in which the temperature in this room is higher than 285 K from the situation in which the temperature is higher than 290 K.

As usual, the relation $\sim_a$ may be defined by means of the set of equivalence classes in $PossWorld$.

In a special case, the relation of epistemological indiscerniblity depends on the knowledge obtained thanks to the behaviour of an agent. For example, if Adam says that he does not know what hat he has, then this utterance may lead Ben and Clark (because of their current epistemic state) to the discernment between the situation that Adam has a red hat and they have white ones and all other (relevant) situations.

The relation of epistemological indiscerniblity is, as the notion of knowledge itself, relative to time: $\sim_{a,t_n}$ is the relation of epistemological indiscerniblity for agent a at time t_n.

It is assumed in our approach that possible epistemic states coincide with the abstraction classes of the epistemological indiscerniblity relation:

$$Epist(a, t_n) = PossWorld/ \sim_{a,t_n} \tag{3}$$

We assume that all changes of epistemic states are caused by the behaviours of the agents—in particular by their utterances, by means of which they expose their (current) epistemic states—and not by their inference processes.

We will now define a number of rules that govern the dynamics of epistemic states. In the hats puzzle the only rule that sets the epistemic states in motion is the following one: *if* agent a (says that) he or she does not know that X holds and in an epistemic state $epist_i(a, t_n)$ a knows that X holds, *then* after the aforementioned utterance that state (i.e. $epist_i(a, t_n)$) is effectively impossible, i.e. we remove its elements from all possible epistemic states of all agents. Formally,

Rule 1 *If (a says that)* $\neg K_{a,t_n}(X)$ *and* $Y \in Epist(a, t_n)$ *and* $K_Y(X)$*, then for every* $a' \in Agent$*,* $Epist(a', t_{n+1}) = \delta_0(Epist(a', t_n), Y)$, where

Definition 7. δ_0 *maps* $Epist \times \bigcup\bigcup Epist$ *into* $Epist$ *and satisfies the following condition:*

$$\delta_0(Epist(a, t_n), X) = \begin{cases} Epist(a, t_n) \backslash \{X\}, & \text{if } X \in Epist(a, t_n), \\ (Epist(a, t_n) \backslash \{Z\}) \cup \{Z \backslash X\} & \text{if } Z \in Epist(a, t_n) \\ & \text{and } X \cap Z \neq \emptyset \\ Epist(a, t_n) & \text{otherwise.} \end{cases}$$

In our conceptual framework we may also define other rules:

Rule 2 *If (a says that)* $K_{a,t_n}(X)$ *and* $Y \in Epist(a, t_n)$ *and* $\neg K_Y(X)$*, then for every* $a' \in Agent$*,* $Epist(a', t_{n+1}) = \delta_0(Epist(a', t_n), Y)$.

Rule 3 *If (a says that)* $\neg \overline{K}_{a,t_n}(X)$ *and* $Y \in Epist(a, t_n)$ *and* $\overline{K}_Y(X)$*, then for every* $a' \in Agent$*,* $Epist(a', t_{n+1}) = \delta_0(Epist(a', t_n), Y)$.

Rule 4 *If (a says that)* $\overline{K}_{a,t_n}(X)$ *and* $Y \in Epist(a, t_n)$ *and* $\neg \overline{K}_Y(X)$*, then for every* $a' \in Agent$*,* $Epist(a', t_{n+1}) = \delta_0(Epist(a', t_n), Y)$.

As one can easily appreciate, these rules presuppose certain degree of rationality of epistemic agents. The rationality manifests itself in the fact that the epistemic condition of those agents, or rather their utterances about their condition, accords with their epistemic states (cf. 4, 5 and 6). Moreover, the rules presuppose that the agents are infallible and (epistemologically) sincere.

If we weaken those assumptions, we may arrive at another notion of rationality. Namely, we assume now that our agents are infallible and (epistemologically) sincere. We do not however assume that the agents' utterances are based on their knowledge of (their) epistemic states. Then when agent a says that:

- he or she knows that X *holds*, then we remove from every epistemic state of every agents all the possible situations that *do not belong* to X,
- he or she knows that X *does not hold*, then we remove from every epistemic state of every agents all the possible situations that *belong* to X.

Formally,

Rule 5 *If (a says that)* $K_{a,t_n}(X)$*, then for every* $a' \in Agent$*,* $Epist(a', t_{n+1}) = \delta_1(Epist(a', t_n), X)$.

Rule 6 *If (a says that)* $\overline{K}_{a,t_n}(X)$*, then for every* $a' \in Agent$*,* $Epist(a', t_{n+1}) = \delta_2(Epist(a', t_n), X)$*,* where

Definition 8. δ_1 *maps* $Epist \times \bigcup\bigcup Epist$ *into* $Epist$ *and satisfies the following condition:*

$$\delta_1(Epist(a,t_n),X) = \begin{cases} (Epist(a,t_n)\backslash\{Z\}) \cup \{Z \cap X\}) & \text{if } Z \in Epist(a,t_n), \\ Epist(a,t_n) & \text{otherwise.} \end{cases}$$

Definition 9. δ_2 *maps* $Epist \times \bigcup\bigcup Epist$ *into* $Epist$ *and satisfies the following condition:*

$$\delta_2(Epist(a,t_n),X) = \begin{cases} (Epist(a,t_n)\backslash\{Z\}) \cup \{Z\backslash X\}) & \text{if } Z \in Epist(a,t_n), \\ Epist(a,t_n) & \text{otherwise.} \end{cases}$$

It seems that the factors that trigger the process of epistemic change are of two kinds: ontological and epistemological. Consider rule 2 once more. The ontological condition of this rule is the fact that agent a says that he or she knows that a certain ontic situation holds (i.e. that $K_{a,t_n}(X)$). On the other hand, the epistemological condition is his or her epistemic state ($Y \in Epist(a, t_n)$) in which the agent cannot know that this situation holds ($\neg K_Y(X)$). We may represent the epistemological conditions of rules for epistemic changes by means of the notion of epistemic state. However, in order to account for the ontological conditions, we distinguish in the set $\wp(ElemSit)$ a subset $AgentBeh$ that collects types (here: sets) of ontic situations that are those conditions. An example of such type may be a set of situations in which agents say that they do not know what hats they have. In general, those conditions may be classified as agents' behaviours, which include also such "behaviours" as being silent (cf. the wisemen puzzle). It is worth noting that, as rules 5 and 6 attest, a rule might not have epistemological conditions.

Let $a \in Agent$. A *rule for epistemic change* ρ is either

1. a mapping $\rho : \bigcup Epist \times AgentBeh \times \bigcup Epist \to \bigcup Epist$ (this condition concerns rules with epistemological conditions) or
2. a mapping $\rho : \bigcup Epist \times AgentBeh \to \bigcup Epist$ (this condition concerns rules without epistemological conditions).

It should be obvious that

1. if $\rho(X, Y, Z) = V$ and $X, Y \in Epist(t_n)$, then $V \in Epist(t_{n+1})$ (for rules with epistemological conditions),
2. if $\rho(X, Y) = V$ i $X \in Epist(t_n)$, to $V \in Epist(t_{n+1})$ (for rules without epistemological conditions).

The set of all such rules will be denoted by "*Rule*". For the sake of brevity, from now on we will consider only rules with epistemological conditions.

Note that all our examples of rules (i.e. rules 1, 2, 3, 4, 5, and 6) actually define multi-agent interactions. Making an utterance of a specific kind, which is described by the relevant ontological trigger of each rules, an agent reveals his or her epistemic condition to other agents who update their own epistemic conditions accordingly. Then, in a sense, our rules describe mutli-agent epistemic actions.

Summarizing, we suggest that one should base his or her solution to a puzzle at stake on such dynamic model of knowledge. In order to obtain the solution, one needs the following input data:

- set Sit,
- temporal sequence $Time = (t_n)$,
- set of epistemic agents $Agent$,
- set of sets of epistemic states of any such agent at the initial moment t_1: $Epist_1 = \{Epist(a, t_1) : a \in Agent\}$,
- function $dist : Time \to \wp(ElemSit)$.

Our function $dist$ is to distribute the agents' behaviours over the set of moments. The behaviours may or may not belong to the elements of $AgentBeh$. When "nothing happens", the value of $dist$ is equal to the empty set.

The evolution of sets of epistemic states is triggered by the ontological conditions, which are determined by function $dist$, according to the accepted rules of epistemic change. This implies that the following condition holds[4]:

$$\exists\rho \in Rule\ \exists X \in AgentBeh \tag{4}$$
$$[\rho(Epist(a, t_n), X, Epist(a', t_n)) = Z \wedge dist(t_n) \cap X \neq \emptyset] \to Epist(a, t_{n+1}) = Z.$$

We also assume that epistemological states change only when a certain rule is triggered:

$$Epist(a, t_{n+1}) \neq Epist(a, t_n) \equiv \tag{5}$$
$$\exists\rho \in Rule \exists X \in AgentBeh \exists Y \in \bigcup Epist[\rho(Epist(a, t_n), X, Y) = Epist(a, t_{n+1})].$$

Of course, it might happen that a rule ρ is triggered vacuously, i.e. for certain a, t_n, X, and Y, it is the case that $\rho(Epist(a, t_n), X, Y) = Epist(a, t_n)$.

After applying the last rule, say at moment t_k ($k \leq n$), we will reach one of the four possible outcomes: (1) in every epistemic state at moment t_k agent a knows that situation X holds (or does not hold), (2) in one epistemic state at moment t_k agent a knows that situation X holds (or does not hold) and in another epistemic state he or she does not know that, (3) in no epistemic state at moment t_k agent a knows that situation X holds (or does not hold) and (4) set $Epist(a, t_{n+1})$ is empty. Only the first case represents the situation in which we (or any other agent, for that matter) are in a position to solve the puzzle at stake. On the other hand, the last situation implies that the puzzle was inconsistent.

The conceptual model, which supports automatic resolutions of a broad class of puzzles, may be seen as a dynamic epistemological model. In general, it is a triple $\langle Sit, Time, Epist\rangle$, which is uniquely determined by the initial assumptions represented by the quadruple $\langle Sit, Epist_1, dist, Rule\rangle$.

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[4] For the sake of simplicity, we assume that function $dist$ does not trigger more than one rule at a time, i.e. we assume that: $[\rho(X_1, Y_1, Z_1) = V_1 \wedge dist(t_n) \cap Y_1 \neq \emptyset] \wedge [\rho(X_2, Y_2, Z_2) = V_2 \wedge dist(t_n) \cap Y_2 \neq \emptyset] \to V_1 = V_2$.

2 Implementation in Prolog

The presented model can be implemented as a program in Prolog in a straightforward way.[5] The user has to introduce the set of parameters: the list of agents, possible values of the attributes (number and colours of hats) and a specification of the perception relation (cf. *sees* relation). On that basis, the initial set of sets of possible epistemic states for all agents is computed. The crucial issue for the puzzle is to find out when an agent knows the colour of his hat and how information about that fact changes other agents epistemic states. We use definition 4 and rule 1 of our model. Definition 4, in the case of the hats puzzle, takes the form of the predicate `knows_his/3` finding all possible epistemic states in which the agent knows the colour of his hat, defined below.

```
knows_his(_,[],[]).
knows_his(A,[H|T1],[H|T2]):-
 unique_in_partition(A,H), !,
 knows_his(A,T1,T2).
knows_his(A,[_|T1],T2):-
 knows_his(A,T1,T2).
unique_in_partition(Agent,Situations):-
 intersection(Situations,Knowledge),
 member((Agent,_),Knowledge).
```

Rule 1 is represented by the predicate `change/4`, in which the function δ_0 defined by the predicate `delta/3` is applied to all epistemic states of all agents.

```
delta([],_,[]).
delta([X|T1],X,T2):-
 delta(T1,X,T2), !.
delta([H1|T1],X,[H2|T2]):-
 intersection(H1,X,I), I\=[],
 subtract(H1,X,H2),
 delta(T1,X,T2), !.
delta([H|T1],X,[H|T2]):-
 delta(T1,X,T2).
```

For example, if one introduces the parameters by the following predicates' definitions:

```
colour(red, 3).
colour(white, 2).
agents([Adam,Ben,Clark]).
sees(Adam,Ben).
sees(Adam,Clark).
sees(Ben,Clark).
```

[5] We present that elements of the program which directly corespond with our model of knowledge ommitting purely technical parts. Full text avialable at http://l3g. Program was tested using SWI-Prolog (ver. 5.6.61).

and ask the following query:

```
?- all_agents_i_p_s(L), change(not_knows,Adam,L,M),
change(not_knows,Ben,M,N),get_result(Clark,N,HatList).
```

the program gives the answer:

```
HatList = [red]
```

The presented model can be applied to a wide range of puzzles concerning knowledge. Just by changing parameters of the program one can solve the hats puzzle with any number of agents and different perception relations. Two of the special cases here are known from literature puzzles: *three wise men* and *muddy children*. With minor changes the program can be also applied to the puzzle *Mr P. and Mr S.* and some puzzles about knights and knaves presented by R. Smullyan.

3 Comparison with other approaches

Knowledge is formally studied in the following frameworks:

(DEL) Dynamic Epistemic Logic
(EBA) Event-Based Approach
(SC) Situation Calculus

DEL has two components, Kripke epistemic structures and dynamic epistemic logic languages to make assertions about them. Kripke structures considered here are of the form $\mathcal{K} = \langle S, Agent, \sim \rangle$, where S is a set of situations (or possible worlds), $Agent$ is a set of rational agents and $\sim$ is a function assigning a set of pairs of situations about which it is said that they are indiscernible to every element of $Agent$. Usually $\sim_a$ are assumed to be equivalence relations. In this approach, knowledge is not expressed directly in $\mathcal{K}$ structures, but syntactically, by formulas with modal operator of epistemic logic of the form $K_a\varphi$— agent a knows that φ [15, 6]. The satisfaction condition for the knowledge formulas in model $\mathcal{M} = \langle \mathcal{K}, v \rangle$ (where $\mathcal{K}$ is the described structure and v is a standard valuation function) and for $s \in S$ is the following:

$$\mathcal{M}, s \models K_a\varphi \iff \forall s' \in S \ (s \sim_a s' \Longrightarrow \mathcal{M}, s' \models \varphi)$$

It says that the formula "agent a knows that φ" is true in situation s (of $\mathcal{M}$) if and only if for all situations which are indiscernible (for agent a) with s, φ is true.

Model of knowledge proposed by "pure" epistemic logic is static, i.e. does not allow for representing a change of agent's knowledge. In order to make it dynamic, dynamic epistemic logics have been created [14]. An example of such logic is a public announcement logic ([11, 4]) which is built by adding to the language of epistemic logic a new operator which enables to express formulas of the form "$[\varphi]\psi$", which are read "after it was publicly and truthfully announced that φ, it is the case that ψ. That formula has the following satisfaction condition in $\mathcal{M}$:

$$\mathcal{M}, s \models [\varphi]\psi \iff \mathcal{M}, s \models \varphi \Longrightarrow \mathcal{M}^{\varphi}, s \models \psi$$

where $\mathcal{M}^{\varphi} = \langle S', A, \sim', v' \rangle$ is characterized by the following conditions:

- $S' = \{s \in S : \mathcal{M}, s \models \varphi\}$,
- for every $a \in Agent$, $\sim'_a = \sim_a \cap (S' \times S')$
- and for every $p \in Atm$, $v'(p) = v(p) \cap S'$.

The second approach, **EBA**, called the event-based approach [3], is more typical for game theory and mathematical economics. Instead of Kripke structures the following structures are in use here: $\langle S, Agent, P \rangle$, where S and $Agent$ are, similarly as in Kripke structures, a set of situations and a set of agents respectively and P is a set of partitions of S for every agent from $Agent$. The event-based approach focuses only on events (and sometimes states of affairs), which are represented by sets of situations. It is said in the approach that event $\mathbf{X}$ takes place in situation s if and only if $s \in \mathbf{X}$.

Knowledge in this approach (cf. [3, Section 2.5]) is modeled by a function $\mathbf{K}_a : 2^S \longrightarrow 2^S$ defined as follows $\mathbf{K}_a(\mathbf{X}) = \{s \in S : P_a(s) \subseteq \mathbf{X}\}$, where

- P_a is partition of S for agent a and $P_a(s)$ is a cell of the partition P_a to which s belongs
- $\mathbf{X}$ is an event or state of affairs.

Thus $\mathbf{K}_a(\mathbf{X})$ is a set of situations in which agent a knows that event $\mathbf{X}$ occurs. It can be said that a knows that $\mathbf{X}$ in situation s if and only if $\mathbf{X}$ holds at every situation $s' \in P_a(s)$.

In [10] we can find another definition of knowledge which is understood as a union of set $\mathbf{K}_a(\mathbf{X})$ and set $\overline{\mathbf{K}}_a(\mathbf{X}) = \{s \in S : P_a(s) \cap \mathbf{X} = \emptyset\}$. $\overline{\mathbf{K}}_a(\mathbf{X})$ is a set of situations in which a knows that $\mathbf{X}$ does not occur.

There is an evident correspondence between the structures of **EBA** and Kripke structures. Every partition P_a can be defined in an obvious way by the indiscernibility relation $\sim_a$ and vice versa by definition:

$$s \sim_a s' \iff P_a(s) = P_a(s')$$

In **SC** [12], knowledge is modeled as a fluent, i.e. a predicate or a function whose truth value may vary. In [9, 13] a fluent $Knows(P, s)$ (P is known in situation s) is defined as follows:

$$Knows(P, s) \iff \forall s'(K(s, s') \Longrightarrow P(s'))$$

where K is accessibility relation between situations. In the scope of this approach, the following issues are considered:

- which actions are knowledge-producing actions and which are not,
- under which conditions carrying out a knowledge-producing action in some situation leads to the successor situation in which something is known (General Positive Effect Axioms for Knowledge) or is not known (General Negative Effect Axioms for Knowledge),

- *successor state axiom* combining General Positive and Negative Effect Axioms for Knowledge, completeness assumption (i.e. an assumption that General Positive and Negative Effect Axioms for Knowledge characterize all the conditions under which an action leads to knowing or not knowing) and unique name axioms,
- frame problem (typically solved by successor state axioms),
- question of knowledge persistence.

Our model share many intuitions which lie behind the frameworks shortly described above and at the same time is distinct form each of them in some aspects. Below we shortly describe some similarities and differences.

- In our approach a set of possible situations (or worlds) is defined, whereas in all listed above frameworks it is taken to be primitive. Possible situation/world in our model is a maximal set of elementary ontic (possible) situations.
- The concept of event/state of affair **X** in **EBA** and **PAL**, for $X \in Sit$, corresponds to the set: $X^{PW} = \{Y \in PossWorld : X \cap Y \neq \emptyset\}$
- For fixed time t_n, $Epist(a, t_n)$ and $epist_i(a, t_n)$ correspond to partition P_a and to some cell of P_a in **EBA** respectively.
- Indiscernibility relations in **PAL** and in our model are isomorphic
- Correspondence between the concept of knowledge in our model and in **EBA** (in static aspect, i.e. without taking into account the interaction between agents) is established by the formula:

$$K_{epist_i(a,t_n)}(X) \iff epist_i(a, t_n) \subseteq \mathbf{K}_a(X^{PW})$$

$$\overline{K}_{epist_i(a,t_n)}(X) \iff epist_i(a, t_n) \subseteq \overline{\mathbf{K}}_a(X^{PW})$$

- In contrast to **PAL** in our model we do not have a distinction between a structure and a language in which the assertions about the structure are made. In our model we express all kinds of intuitions in one framework.
- The concepts of knowledge in our model and in **PAL** are similar. Having in mind the satisfaction condition for knowledge operator in **PAL** we can define knowledge in our model in **PAL**-like style

$$K_{epist_i(a,t_n)}(X) \iff \forall Y \in PossWorld\ (Y \in epist_i(a, t_n) \Longrightarrow Y \in X^{PW})$$

$$\overline{K}_{epist_i(a,t_n)}(X) \iff \forall Y \in PossWorld\ (Y \in epist_i(a, t_n) \Longrightarrow Y \notin X^{PW})$$

- The idea of reducing the agent's ignorance by making proper changes in the sets of indiscernible situations as a result of receiving new information is shared by **PAL** and our model. However, what makes a difference is the generality of our approach. In our model we consider many kinds of ontological and epistemological factors that trigger the process of epistemic change, whereas in **PAL** we just have announcements as triggers.
- We share with **SC** the interest in the issues which were listed above. In our model we have the condition of knowledge persistence (see formula 5) and our rules imitate the General Positive Effect Axioms for Knowledge. Our concept of (maximal) situation is very similar to McCarthy's view on situations as snapshots and is far different from Reiter's situation as a sequence of actions.

4 Further work

Our plan for future work includes:

- Development of our model of epistemic interactions by incorporating more sophisticated phenomena such as: shared and common knowledge or non-verbal behavior. Another line of development would take into account more features of the real-world knowledge. For example, the relation of indiscernability, in most of the realistic scenarios, is not transitive.
- Investigating computational properties of our model when applied in more complex epistemic situations.
- More detailed work concerning the relation of our model to other frameworks dealing with knowledge.

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